

Connect 4 Maths

By Cal Henderson

I get an email like this from time to time:

Im doing some gsce coursework at the moment and our aim is to investigate the number of wiining lines in the game connect 4.we have to find a series of formulas. Do you have any info which would help me?

So here's my stock answer:

Let your grid be x by y squares large
Let the length of a solution be n squares

(So in a standard game, $x=7$, $y=6$, $n=4$)

horizontal lines = $y * (x-n)$ (NOTE: while $x-n > 0$)
vertical lines = $x * (y-n)$ (NOTE: while $y-n > 0$)
diagonal lines = $2 * (x-n) * (y-n)$ (NOTE: while $x-n > 0$ and $y-n > 0$)

total lines = $(y * (x-n)) + (x * (y-n)) + (2 * (x-n) * (y-n))$

If $x-n$ or $y-n$ are 0 or less, then the board is too small to fit a horizontal or vertical winning line in. Obviously this means the game is unplayable (e.g. playing connect 4 on a 3 by 3 grid) but is worth remembering when working out the formulas.

Here's a bit of an extension:

Let your grid be x by y squares large
Let the solution line be a by b squares large (for solution rectangles)

(So in a standard game, $x=7$, $y=6$, $a=4$, $b=1$)

horizontal lines = $(y-(b-1)) * (x-a)$
vertical lines = $(x-(b-1)) * (y-a)$
diagonal lines = $2 * (x-(a+b-1)) * (y-(a+b-1))$

total lines = $((y-(b-1)) * (x-a)) + ((x-(b-1)) * (y-a)) + (2 * (x-(a+b-1)) * (y-(a+b-1)))$

If you want to start getting fancy, consider 3 dimensional grids. Infact, you can quite easily find a general solution for a grid that's $a_1, a_2, a_3 \dots a_n$ in size, with a winning line of $b_1, b_2, b_3 \dots b_n$ in size. I'll leave that one for you to work out ;)

(end of article)